1. Set Theory

Q.1.1.1.1 Prove that $\phi \subseteq A$, where ϕ is the empty set and A is an arbitrary set.
Answer: There exists no element x such that $x \in \phi$ and $x \notin A$. So, A is a superset of ϕ.

Q.1.1.1.2 Prove that the power set of S has 2^n elements, where $|S| = n$.
Answer: The cardinality of set S is n. The number of subsets each containing k elements is ${}^nC_k$. Total number of subsets of $S = \Sigma_{k=0}^{n} {}^nC_k = (1+1)^n = 2^n$. It follows from the fact that $(1+x)^n = {}^nC_0x^n + {}^nC_1x^{n-1} + \cdots + {}^nC_{n-1}x + {}^nC_n$. Then put $x = 1$.

Q.1.1.1.3 For any two sets A and B, prove that $A\Delta B = (A \cup B) - (A \cap B)$, where $A\Delta B = \{x : x \in A - B \text{ or } x \in B - A\}$.
Answer: Let $x \in (A \cup B) - (A \cap B) \iff x \in A \cup B$ and $x \notin A \cap B$
$\iff (x \in A$ or $x \in B)$ and $x \notin (A \cap B)$
$\iff (x \in A$ and $x \notin (A \cap B))$ or $(x \in B$ and $x \notin (A \cap B))$
[applying distributive property]
$\iff (x \in A - B)$ or $(x \in B - A)$
$\iff x \in (A - B) \cup (B - A)$

Q.1.1.1.4 Let A and B be two sets. Prove that $(A \cup B)' = A' \cap B'$.
Answer: Let $x \in (A \cup B)' \iff x \notin A \cup B \Rightarrow x \notin A$ and $x \notin B$
$\iff x \in A'$ and $x \in B' \iff x \in A' \cap B'$
Therefore, $(A \cup B)' \subseteq A' \cap B'$ (1)
Again, let $y \in A' \cap B' \iff y \in A'$ and $y \in B'$
$\iff y \notin A$ and $y \notin B \implies y \notin A \cap B \iff y \notin (A \cap B)'$
Therefore, $A' \cap B' \subseteq (A \cap B)'$ (2)
From (1) and (2), we get $(A \cup B)' = A' \cap B'$.

Q.1.1.1.5 Show that the strings of 0s and 1s of arbitrary length are uncountable infinite.
Answer: If possible let, the strings of 0s and 1s of arbitrary length are countable. So, we can have a mapping between the set of natural numbers and the set strings of arbitrary length. Table 1 gives a part of this mapping.
Now let us consider a new string by interchanging 0s and 1s in the diagonal elements, i.e., the new string formed is 10101..., see Table 1. This string does not match any string listed in the table. So, this set is uncountably infinite.

Srl No	String
1	001101...
2	011101...
3	000000...
4	001111...
5	111000...
...	...

Table 1: All strings of 0's and 1's

Q.1.1.1.6 For any two arbitrary sets A and B, $(A - B) \cap B = \phi$ holds true. Prove it.
Answer: Let us assume that $(A - B) \cap B \neq \phi$. So, there exists an element $x \in (A - B) \cap B \Longleftrightarrow x \in A - B$ and $x \in B$
$\Longleftrightarrow (x \in A$ and $x \notin B)$ and $x \in B \Longleftrightarrow x \in A$ and $(x \notin B$ and $x \in B)$
But there is no element that satisfies $x \in B$ and $x \notin B$.
Therefore, $(A - B) \cap B = \phi$.

Q.1.1.1.7 Prove that $A \times (B \cup C) = (A \times B) \cup (A \times C)$, for arbitrary sets A, B, and C.
Answer: Let $(x, y) \in A \times (B \cup C) \Longleftrightarrow x \in A$ and $y \in B \cup C$
$\Longleftrightarrow x \in A$ and $(y \in B$ or $y \in C) \Longleftrightarrow (x \in A$ and $y \in B)$ or $(x \in A$ and $y \in C)$
$\Longleftrightarrow (x, y) \in (A \times B)$ or $(x, y) \in (A \times C) \Longleftrightarrow (x, y) \in (A \times B) \cup (A \times C)$
The result follows.

Q.1.1.1.8 Show that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$, where $n(X)$ is the number of element in set X. (Assume that sets A and B are finite)
Answer: Note that $A \cap B \subseteq A$ and $A \cap B \subseteq B$.
So, $A = (A - A \cap B) \cup (A \cap B)$ and $B = (B - A \cap B) \cup (A \cap B)$
Therefore, $n(A) = n(A - (A \cap B)) + n(A \cap B)$ and
$n(B) = n(B - (A \cap B)) + n(A \cap B)$
[Since $(A \cap B) \cap (A - (A \cap B)) = \phi$ and $(A \cap B) \cap (B - (A \cap B)) = \phi$]
$\therefore n(A - (A \cap B)) = n(A) - n(A \cap B)$
and $n(B - (A \cap B)) = n(B) - n(A \cap B)$ (3)
Again, $A \cup B = ((A - (A \cap B)) \cup (B - (A \cap B)) \cup (A \cap B))$
Therefore, $n(A \cup B) = n(A - (A \cap B)) + n(B - (A \cap B)) + n(A \cap B)$
[Since, $(A - (A \cap B)) \cap (B - (A \cap B)) = \phi$ and $(A \cap B) \cap (B - (A \cap B)) = \phi$ and $(A - (A \cap B)) \cap (A \cap B) = \phi$]
Therefore, $n(A \cup B) = n(A) - n(A \cap B) + n(B) - n(A \cap B) + n(A \cap B)$
$= n(A) + n(B) - n(A \cap B)$, using (3)

Q.1.1.1.9 A survey on a sample of 25 news paper reading people was conducted. The following facts were found:
16 people read Times of India
12 people read Hindustan Times
11 people read Indian express

8 people read both Times of India and Hindustan Times
8 people read Times of India and Indian Express
3 people read both Hindustan Times and Indian Express
1 people read all the three papers
Find the number of people who read (a) only Indian Express, (b) Times of India but not Hindustan Times, (c) Hindustan Times and Indian Express but not Times of India, (d) only one paper, (e) at least two papers, (f) none of the papers, (g) at least one paper.
Answer:

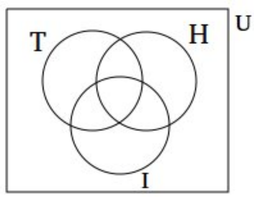

Figure 1: Venn diagram for people who read Times of India, Hindustan Times and Indian Express

T: Set of people who read Times of India
H: Set of people who read Hindustan Times
I: Set of people who read Indian Express

$$n(T \cup H \cup I) = n(T) + n(H) + n(I) - n(T \cap H) - n(T \cap I) - n(H \cap I) + n(T \cap H \cap I)$$
$$= 16 + 12 + 11 - 8 - 8 - 3 + 1 = 36 - 16 + 1 = 40 - 19 = 21 \quad (4)$$

(a) The number of people who read only Indian Express $= n(I) - [n(T \cap I) - n(T \cap H \cap I)] - [n(H \cap I) - n(T \cap H \cap I)] - n(T \cap H \cap I) = n(I) - n(T \cap I) - n(H \cap I) + n(T \cap H \cap I) = 11 - 8 - 3 + 1 = 1$
(b) The number of people who read Times of India but not Hindustan Times $= n(T) - n(T \cap H) = 16 - 8 = 8$
(c) The number of people who read Hindustan Times and Indian Express but not Times of India $= n(H \cap I) - n(T \cap H \cap I) = 3 - 1 = 2$
(d) The number of people who read only one paper = The number of people who read at least one paper - (The number of people who read at least two paper+ number of people who read 3 papers) $= n(T) + n(H) + n(I) - [n(T \cap H) + n(T \cap I) + n(H \cap I) + n(T \cap H \cap I)]$ = 16 + 12 + 11 - [8 + 8 + 3 + 1] = 39 - 20 = 19
(e) The number of people who read at least two papers $= n(T \cap I) - n(T \cap H \cap I) + n(T \cap H) - n(T \cap H \cap I) + n(H \cap I) - n(T \cap H \cap I) + n(T \cap H \cap I)$
$= n(T \cap I) + n(T \cap H) + n(H \cap I) - 2 \times n(T \cap H \cap I) = 8 + 8 + 3 - 2 = 17$
(f) The number of people who do not read any news paper
$= n(U) - n(T \cup H \cup I) = 25 - 21 = 4$ [U is the universal set]
(g) The number of people who read at least one paper is = 21 [from (4)]

2. Relation

Q.1.1.2.1 Let $A = \{1, 2, 3, 4, 5\}$, and $B = \{6, 7, 8, 9, 11\}$. Consider a relation R such that $(a, b) \in R$ if an only if a and b are both prime numbers, $a \in A, b \in B$. Find R. Also find domain and range of R.
Answer: $R = \{(2, 7), (2, 11), (3, 7), (3, 11), (5, 7), (5, 11)\}$
The domain of $R = \{2, 3, 5\}$, and the range of $R = \{7, 11\}$

Q.1.1.2.2 Does a non-symmetric relation become antisymmetric?
Answer: Let $R = \{(1, 2), (2, 1), (2, 3)\}$ be a relation on $A = \{1, 2, 3\}$. R is neither symmetric nor anti-symmetric. R is not symmetric since $(3, 2) \notin R$, and $(2, 3) \in R$. R is not anti-symmetric since $1 \neq 2$ and $(1, 2) \in R$, $(2, 1) \in R$. Let $R1 = \{(1, 1), (3, 3)\}$ be a relation on $A = \{1, 2, 3\}$. Here $R1$ is both symmetric and anti-symmetric.

Q.1.1.2.3 Write the differences between a relation and a function.
Answer: Some differences between a relation and a function are given as follows:
1. Every function is a relation but every relation is not a function.
2. If R is a relation from A to B then domain of R may be a subset of A. But if f is a function from A to B, then domain of f is equal to A.
3. In a relation from A to B, an element of A may be related to more than one element in B. Also, there may be some elements in A which may not be related to any element in B. But in a function from A to B, each element of A must be associated with one and only one element of B.

Q.1.1.2.4 $R = \{(x, y) : x - y$ is divisible by $5\}$ is a relation on $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Show that R is an equivalence relation. Write the matrix for R and sketch its graph.
Answer: For every $a \in A, a - a = 0$ is divisible by 5, i.e., aRa. So, R is reflexive.
For $a, b \in A$, if $a - b$ is divisible by 5, then $b - a$ is also divisible by 5, i.e., $aRb \Rightarrow bRa$. So, R is symmetric.
For $a, b, c \in A$, if $a - b$ and $b - c$ are divisible by 5, then a - c = (a - b) + (b - c) is also divisible by 5.
So, aRb and $bRc \Rightarrow aRc \; \forall \; a, b, c \in A$. So R is transitive.
Therefore, R is an equivalence relation.
Now $R = \{(1, 1), (1, 6), (2, 2), (2, 7), (3, 3), (3, 8), (4, 4), (5, 5), (6, 6), (7, 7), (8, 8), (6, 1), (7, 2), (8, 3)\}$

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

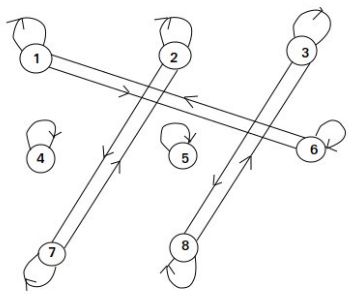

Figure 11: Graph expressing relation R

Q.1.1.2.5 Let $S = \{2, 4, 6, 8, 10\}$ and let R be the relation "x divides y" on S, i.e., xRy iff there exists an integer z such that $x \times z = y$
(i) Find R, (ii) Draw the graph for R, (iii) Find R^{-1} and express it in words.
Answer: (i) 2 divides 2, 4, 6, 8 and 10. So, (2, 2), (2, 4), (2, 6), (2, 8), (2, 10) $\in R$. Again (6, 6), (8, 8), (10, 10) $\in R$
4 divides 4, 8. So, (4, 4), (4, 8) $\in R$
So, R = {(2, 2), (2, 4), (2, 6), (2, 8), (2, 10), (4, 4), (4, 8), (6, 6), (8, 8), (10, 10)}
(ii)

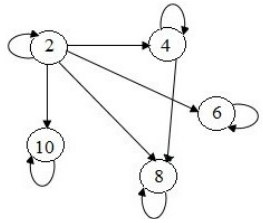

Figure 12: Graph for the given relation R

(iii) We reverse the ordered pairs of R to obtain R^{-1}.
$R^{-1} = \{(2,2),(4,2),(4,4),(6,2),(6,6),(8,2),(8,4),(8,8),(10,2),(10,10)\}$
R^{-1} can be described as "x is a multiple of y".

Q.1.1.2.6 Let R be a relation on the set of natural numbers N and defined by the equation $2x + 3y = 16$.
(i) Write R as a set of ordered pairs.
(ii) Find domain (R) and range (R).
(iii) Find the composition relation $R \circ R$.
Answer: (i) $R = \{(x,\ y)\colon 2x + 3y = 16\}$, i.e., $R = \{(5,\ 2),\ (2,\ 4)\}$
(ii) domain$(R) = \{2,\ 5\}$, range$(R) = \{2,\ 4\}$
(iii) $R \circ R = \{(5, 4)\}$

Q.1.1.2.7 Let $R = \{(a,\ b),\ (a,\ c),\ (c,\ b)\}$ be a relation on $X = \{a,\ b,\ c\}$. Find transition closure of R on X.
Answer: Transitive closure of R on X, $R^+ = R \cup R^2 \cup R^3 \cup R^4 \cup \ldots$
Now, $R^2 = R \circ R = \{(a, b)\}$
$R^3 = R \circ R^2 = \phi$ (empty set)
So, $R^n = \phi$, for $n > 3$.
$R^+ = \{(a, b), (a, c), (c, b)\} = R$

Q.1.1.2.8 Consider a relation R on $A = \{1,\ 2,\ 3\}$ such that aRb if $a \geq b; a, b \in A$. Find the members of R. Check whether R is reflexive, symmetric, antisymmetric and transitive.
Answer: Here, $R = \{(1,\ 1),\ (2,\ 1),\ (2,\ 2),\ (3,\ 1),\ (3,\ 2),\ (3,\ 3)\}$
(i) R is reflexive, since $(a, a) \in R$, for $a \in A$
(ii) R is not symmetric, since $(2, 1) \in R$ but $(1, 2) \notin R$
(iii) R is antisymmetric, since $(a, b) \in R$ and $a \neq b$ then $(b, a) \notin R$
(iv) R is a transitive, since $(3, 2) \in R$ and $(2, 1) \in R$ then $(3, 1) \in R$.

Q.1.1.2.9 Let A be the set of all positive integers. R is a relation on A such that a divides b for $a, b \in A$. Analyze R.
Answer: (i) R is reflexive, since $(a, a) \in R$
(ii) R is not symmetric since $(a, b) \in R$ does not always mean that b divides a.
(iii) R is transitive since a divides b, b divides c implies a divides c.
(iv) a divides $-a$ and $-a$ divides a. But, $a \neq -a$, for $a \in A$. Thus, R is not antisymmetric.

Q.1.1.2.10 Let T be the set of all triangles. Let R be the relation "a is similar to b", for $a, b \in T$. Show that R is an equivalence relation.
Answer: For $a, b, c \in T$, the following observations are made:
R is reflexive, since every triangle is similar to itself.
If a is similar to b and b is similar to c then a is similar to c. So, R is transitive.
If a is similar to b then b is similar to a. So, R is symmetric. Thus, R is an equivalence relation.

3. Function

Q.1.1.3.1 Consider the domain and codomain of a function $f = \{(1,\ a),\ (3,\ c),\ (4,\ c)\}$ are $A = \{1,\ 2,\ 3,\ 4\}$ and $B = \{a,\ b,\ c\}$, respectively. Discuss the type and nature of the function.
Answer: Note that $2 \in A$, but it does not correspond to any value in B. So, f is not a function.

Q.1.1.3.2 Let $A = \{1,\ 2,\ 3,\ 4\}$ and $B = \{a,\ b,\ c,\ d\}$ be the domain and co-domain of the functions $f(x) = d,\ x \in A$, respectively. Judge whether f is a function. Find its range / image set.
Answer: $f(x)$ is a function, since every element of domain has a unique image in B. It is an example of a constant function. The range / image set is $\{d\}$.

Q.1.1.3.3 What is identity function?
Answer: Let $f : A \to A$ be a function defined by $f(a) = a, \forall a \in A$. f is called an identity function.

Q.1.1.3.4 Let $f = \{(a,\ b),\ (b,\ a),\ (c,\ c)\}$ with domain = co-domain = $\{a,\ b,\ c\}$. Find f^{-1}, f^2.
Answer: $f(a) = b, f(b) = a, f(c) = c$. Therefore , $f^{-1}(b) = a, f^{-1}(a) = b, f^{-1}(c) = c$. Thus , $f^{-1} = \{(b,a),(a,b),(c,c)\}$.
$f^2(a) = f(f(a)) = f(b) = a$. $f^2(b) = f(f(b)) = f(a) = b$. $f^2(c) = f(f(c)) = f(c) = c$. Thus , $f^2 = \{(a,a),(b,b),(c,c)\}$.

Q.1.1.3.5 Prove that $\lfloor 2x \rfloor = \lfloor x \rfloor + \lfloor x + \frac{1}{2} \rfloor, x$ is a real number.
Answer: We shall prove it for positive number. Proof becomes similar for negative number.
Let $x = y + \epsilon$, where y is an integer, and $0 \le \epsilon < 1$
Then , $2x = 2y + 2\epsilon$.

$$\lfloor 2x \rfloor = \begin{cases} 2y+1, & \text{when } 1 \le 2\epsilon < 2, \text{ i.e., when } \frac{1}{2} \le \epsilon < 1, \\ 2y, & \text{when } 0 \le 2\epsilon < 1, \text{ i.e., when } 0 \le \epsilon < \frac{1}{2} \end{cases}$$

Then, $\lfloor x \rfloor = \lfloor y + \epsilon \rfloor = y$.

$$\lfloor x + \tfrac{1}{2} \rfloor = \lfloor y + \epsilon + \tfrac{1}{2} \rfloor = \begin{cases} y+1, & \text{when } \frac{1}{2} \le \epsilon < 1 \\ y, & \text{when } 0 \le \epsilon < \frac{1}{2} \end{cases}$$

So, the result follows.

Q.1.1.3.6 Explain the concept of primitive recursive function.
Answer: A function f is recursive, if it can be obtained from the following primitive functions by applying recursion finite number of times.
(i) $Z(x) = 0$
(ii) $S(x) = x + 1$

(iii) $\bigcup_i^n(x_1, x_2, \ldots, x_n) = x_i$
where, $x, x_i \in N$, the set of natural numbers.

Q.1.1.3.7 Show that $f(x,\ y) = x\ +\ y\ +\ 2$ is primitive recursive.
Answer: $f(x, y+1) = x+(y+1)+2 = x+y+2+1 = f(x, y)+1 = S(f(x, y))$.
$f(x, 0) = x + 2 = S(x + 1) = S(S(x)) = S(S(\cup_1^1(x)))$.
$f(x, y + 1) = S(f(x, y)) = S(\cup_3^3(x, y, f(x, y))$.
f is a primitive recursive.

Q.1.1.3.8 Check whether the function $f(x) = \lceil \frac{1}{2} \rceil$ from Z to Z is (i) one-to-one, (ii) onto, where Z is the set of integers.
Answer: $f(0) = \lceil \frac{0}{2} \rceil = 0, f(1) = \lceil \frac{1}{2} \rceil = 1, f(-1) = \lceil -\frac{1}{2} \rceil = 0,$
$f(2) = \lceil \frac{2}{2} \rceil = 1, f(-2) = \lceil -\frac{2}{2} \rceil = -1$
...
Some values in the domain correspond to a particular value in the co-domain. For example, 0 correspond to 0 and -1 correspond to 0. Therefore, f is not one-to-one. Also, we find that every value in the co-domain has some corresponding values in the domain. For example, 0 in the co-domain has two corresponding values 0 and -1 in the domain. Therefore, f is an onto function.

Q.1.1.3.9 Prove the following inequality: $x - 1 < \lfloor x \rfloor \leq x \leq \lceil x \rceil < x + 1$
Answer: Let $x = n + \epsilon$, where n is a positive integer and $0 \leq \epsilon < 1$.
If $\epsilon = 0$ then $\lfloor x \rfloor = x = \lceil x \rceil = n$.
In this case, $x - 1 = n - 1 < n = x$ and $x + 1 = n + 1 > n = x$.
The above inequality is true for $\epsilon = 0$.
If $0 < \epsilon < 1$ then $\lfloor x \rfloor = n$ and $\lceil x \rceil = n + 1$.
In this case, $\lfloor x \rfloor = n < n + \epsilon = x < n + 1 = \lfloor x \rfloor$
$x - 1 = n - 1 + \epsilon < \lfloor x \rfloor$ and $x + 1 = n + 1 + \epsilon > \lfloor x \rfloor$
The above inequality is true for $0 < \epsilon < 1$. The proof follows in a similar way when n is a negative integer.

Q.1.1.3.10 Give an example function from Z to Z^+ for each of the following cases: (i) one-to-one, but not onto, (ii) onto, but not one-to-one, (iii) one-to-one and onto, (iv) neither one-to-one nor onto.
Answer: (i) $f(x) = \begin{cases} 5x + 1, & \text{if } x \geq 0 \\ -5x + 2, & \text{if } x < 0 \end{cases}$
There are following three cases, for $x_1, x_2 \in Z$.
Case 1: When $x_1, x_2 \geq 0$ and $x_1 \neq x_2$ then the mapped values $5x_1 + 1$ and $5x_2 + 1$ are different. So, it is one-to-one.
Case 2: When $x_1 < 0$ and $x_2 \geq 0$ then the mapped values $-5x_1 + 2$ and $5x_2 + 1$ are different. So, it is one-to-one.
Case 3: When $x_1, x_2 < 0$ then the mapped values $-5x_1 + 2$ and $-5x_2 + 2$ are different. So, it is one-to-one.
$2 \in Z^+$. But there is no $x \in Z$ such that $f(x) = 2$. So, it is not onto.
(ii) $f(x) = \ |x\ | + 1$

More than one values map into the same value in the co-domain. For example, $f(-1) = f(1) = 2$. Therefore, it is not one-to-one. But, every member in the co-domain has at least one corresponding value in the domain. In fact, every $y \in R^+$ corresponds to two values in the domain viz., the same value and its negation. So, it is onto.

(iii) $f(x) = \begin{cases} 2x+1, & \text{if } x \geq 0 \\ -2x, & \text{if } x < 0 \end{cases}$

See the proof techniques as discussed in (i) and (ii).

(iv) $f(x) = x^2 + 5$

Follow the techniques as stated in (i) and (ii).

Q.1.1.3.11 Let $g : A \to B$ and $f : B \to C$ are one-to-one functions. Show that $f \circ g$ is one-to-one.

Answer: $g : A \to B$ is an one-to-one function. This implies that $g(x) \neq g(y)$ for $x \neq y$, x, $y \in A$.

Also, $f : B \to C$ is one-to-one function. $f(a) \neq f(b)$, for $a \neq b, a, b \in B$ such that $a = g(x), b = g(y)$.

$f(g(x)) \neq f(g(y))$ for $x \neq y, x, y \in A$. $f \circ g$ is one-to-one.

Q.1.1.3.12 Find the domain and range of a function that assigns the number of bits left over when a bit string is split into bytes, 1 byte = 8 bits.

Answer: Domain = set of bit string. Range = {0, 1, 2, 3, 4, 5, 6, 7}.

Q.1.1.3.13 Check whether the following function is bijection from R to R.

$f(x) = x^3$, $x \in R$.

Answer: There are two conditions to be satisfied.

(i) $f(x)$ is one-to-one, (ii) $f(x)$ is onto.

(i) For each $x \in R, f(x)$ gives a unique value in R. If $x \neq y$ then $x^3 \neq y^3, x, y \in R$. So , f is one-to-one

(ii) Again, for every value of $x^3 \in R$, there is a unique value $x \in R$. If $x^3 \neq y^3$ then $x \neq y, x^3, y^3 \in R$. So, f is onto.

Therefore, f is a bijection.

Q.1.1.3.14 Let $f(x) = \lfloor x \rfloor$. Find pre-image set if the range set (i) $\{x \mid 0 < x < 1\}$, (ii) $\{-2, -1, 0, 1, 2\}$.

Answer: (i) There is no value in the domain set that correspond to any value in $\{x \mid 0 < x < 1\}$. Pre-image set = { } = ϕ.

(ii) Each value in $\{x \mid -2 \leq x < -1\}$ correspond to -2 in the co-domain.

Each value in $\{x \mid -1 \leq x < 0\}$ correspond to -1 in the co-domain.

Each value in $\{x \mid 0 \leq x < 1\}$ correspond to 0 in the co-domain.

Each value in $\{x \mid 1 \leq x < 2\}$ correspond to 1 in the co-domain.

Each value in $\{x \mid 2 \leq x < 3\}$ correspond to 2 in the co-domain.

Pre-image set = $\{x \mid -2 \leq x < -1\} \cup \cdots \cup \{x \mid 2 \leq x < 3\} = \{x \mid -2 \leq x < 3\}$